Number of spaces, $n_s = 3$ spaces

Frequency, f = 20Hz

Period time,
$$T = \frac{1}{f}$$

$$T = \frac{1}{20}$$

$$T = 0.05$$

Time taken, \mathbf{t} =Number of spaces, \mathbf{n}_s ×Period time, T Time taken, \mathbf{t} =3(0.05)

t=0.15s

ii) average speed

Distance covered =
$$10 \text{cm} = 0.1 \text{m}$$

Average speed, $v = \frac{\text{Distance}}{\text{Time taken}}$
 $v = \frac{0.1}{0.15}$
 $v = 0.67 \text{ms}^{-1}$

Example:

A trolley is pulled from rest with a constant force down an inclined plane. The trolley pulls a tape through a ticker timer vibrating at 50Hz. The following measurements were made from the tap.

Distance between 16^{th} dot and 20^{th} dot= $\mathbf{d_1}$ =20cm Distance between 20^{th} dot and 30^{th} dot = 34cm Distance Q between 30^{th} dots and 40^{th} dot = 48cm Distance between 40^{th} dot and 50^{th} dot = $\mathbf{d_2}$ =62cm

Calculate the acceleration of the trolley.

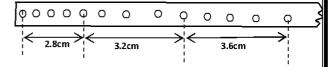
Solution

Acceleration, $a = \frac{\text{change in velocity}}{\text{Time for the change}}$

$u = \frac{d_1}{t_1}$	Acceleration, $a = \frac{v-u}{t_3}$
$u = \frac{0.2}{0.08}$	Acceleration, $a = \frac{3.1-2.5}{0.6}$
$u = 2.5ms^{-1}$	Acceleration, a = 1.0 ms^{-2}
$d_2 = 62cm$ 62	
$d_2 = \frac{100}{100}$ $d_2 = 0.62m$	

Exercise

- 1. A paper tape dragged through a ticker timer by a trolley has the first ten dots covering a distance of 4cm and the next ten dots covering a distance of 7cm. If the frequency of the ticker timer is 50Hz, calculate the acceleration of the trolley.(Ans:=75cms⁻² or 0.75ms⁻²)
- 2. The ticker timer below was pulled by a decelerating trolley. The tape consists of 3 five dot spaces and the frequency of the timer is 50Hz.



Exercise: See UNEB

Exercise: See GIVEB	
2003.Qn.26	2001.Qn.25
1998.Qn.1(b)	2006.Qn.9

1: 11. CIRCULAR MOTION

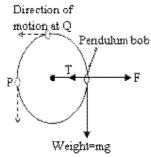
Circular motion is motion in which a body moves in a circle about a fixed point.

For a body moving in a circle;

- ✓ Its direction and velocity are constantly changing.
- ✓ It has an acceleration called centripetal acceleration.

 $t_1 = 0.08s$

✓ It has a force called Centripetal force acting towards the centre of the circular path.



T=Tension in the string which produces the centripetal force

Note: When the object is released, it moves such that the direction of motion at any point is along a tangent to the circular path.

Forces acting on the body describing circular motion.

- (i) **Tension:** Force acting towards the centre of the circular path. It provides the centripetal force.
- (ii) **Centripetal force**: Force acting towards the centre of the circular path.
- (iii) **Centrifugal force**: Force acting away from the centre of the circular path.
- (iv) **Weight:** Force acting vertically down wards towards the centre of the earth.

Examples of circular motion

- -Pendulum bob tied to a string whirled in a vertical or horizontal plane
- -Planetary motion etc

Exercise: See UNEB 1999 Paper II Qn.1

1: 12. NEWTON'S LAWS OF MOTION

These are three laws that summarize the behavior of particles in motion.

1:12:1. Newton's First Law of motion

Newton's first law of motion states that a body continues in its state of rest or uniform motion in a straight line unless acted upon by an external force.

Inertia

Inertia is the reluctance of a body to move, when at rest or to stop when moving.

Thus, when a force acts on a body, the body;

- ✓ Starts or stops moving.
- ✓ Increases or reduces speed depending on the direction of the force.
- ✓ Changes direction of motion.

1:12:2. Newton's second law of motion

Newton's second law states that the rate of change in momentum is directly proportional to the force acting on the body and takes place in the direction of the force.

$$F\:\alpha\frac{mv-mu}{t} \Longleftrightarrow F\:\alpha\:m\Big(\!\frac{v-u}{t}\!\Big) \Longleftrightarrow F\:\alpha\:ma \Longleftrightarrow F=k\:ma$$

When we consider a force of 1N, mass of 1kg and acceleration of 1ms^{-2} , then, **k=1**. Therefore;

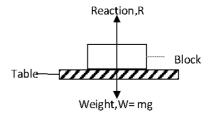
$$F = ma$$

A newton; Is the force which acts on a mass of 1kg to produce an acceleration of 1ms^{-2} .

1:12:3. Newton's third law of motion

It states that action and reaction are equal but opposite.

When a body, A exerts a force on body B, body B also exerts an equal force in the opposite direction.



The block exerts a weight, W= mg on the table and the table also exerts an equal reaction R on the block. R= mg, so that the net force on the block is zero and therefore there is no vertical motion.

Applications of Newton's third law of motion

(a) Rockets and jets

Rockets and jet engines are designed to burn fuel in oxygen to produce large amounts of exhaust gases.

These gases are passed backwards through the exhaust pipes at high velocity (large momentum).

This in turn gives the Rocket or jet a high forward momentum which is equal but opposite to that of the exhaust gases.

$$m_q v_q = -m_R v_R$$

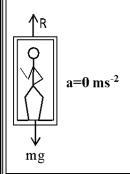
Where $m_g v_g$ is the momentum of the exhaust gases, and $m_R v_R$ is momentum of the Rocket.

(b) Motion in the lift

Consider a person of mass m standing in a lift, when the;

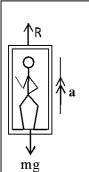
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i) Lift is stationary or moving with uniform velocity



The person exerts a weight, mg on the lift and at the same time, the lift exerts a reaction, R, on the person. **R=mg**.

ii) Lift is moving upwards with acceleration, a.



In this case, three forces act on the lift. i.e, the resultant accelerating force (ma), the weight, (mg) and the normal reaction or Apparent weight (R).

Accelerating force = Net force ma = R - mg

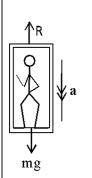
$$R = mg + ma$$

$$R = m (g+a)$$

Thus, the reaction on the person (apparent weight, R) is greater than the actual weight of the person, mg.

This is why one feels **heavier** when the lift is just beginning its upward journey.

iii) Lift is moving down wards with acceleration, a.



In this case, the resultant accelerating force (ma), and the weight, (mg) act down wards. The normal reaction or Apparent weight (R) act upwards.

Accelerating force = Net force

$$ma = mg - R$$

$$R = mg - ma$$

$$R = m(g - a)$$

Thus, the reaction on the person (apparent weight, R) is less than the actual weight of the person, mg.

This is why one feels **lighter** when the lift is just beginning its downward journey.

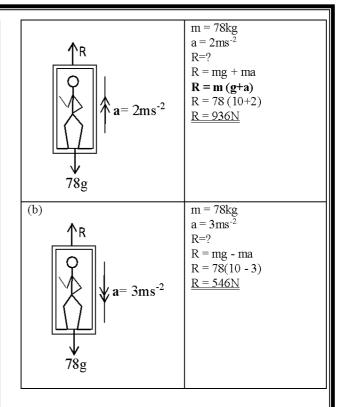
Example:1

A person of mass 78kg is standing inside an electric lift. What is the apparent weight of the person if the;

- d) Lift is moving upwards with an acceleration of 2ms⁻²?
- e) Lift is descending with an acceleration of 2ms⁻²?

Solution

(a)



1: 12: 4. COLLISIONS AND MOMENTUM

Linear Momentum:

Momentum is the product of mass and its velocity.

$$\binom{\text{Linear Momentum}}{\text{of a body}} = \binom{\text{Mass of}}{\text{the body}} \times \text{Velocity}$$

Impulse:

Impulse is the change in the momentum of a body.

$$Impulse = mv - mu$$

Impulse can also be defined as the product of force and time of impact.

From Newton's second law of motion,

$$F = \frac{mv - mu}{t} \iff Ft = mv - mu$$

$$Impulse = Ft = mv - mu$$

The S.I unit of momentum and impulse is **Kgms**⁻¹ **Note**: Momentum and impulse are vector quantities.

Principle of conservation of momentum

It states that when two or more bodies collide, the total momentum remains constant provided no external force is acting.

It states that when two or more bodies collide, the total momentum before collision is equal to the total momentum after collision.

Suppose a body of mass m_1 moving with velocity u_1 collides with another body of mass m_2 moving with velocity u_2 . After collision, the bodies move with velocities v_1 and v_2 respectively, then;

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

Types of collisions

✓ Elastic collision

Elastic collision is the type of collision whereby the colliding bodies separate immediately after the impact with each other and move with different velocities.

In short, for elastic collision,

✓ Inelastic collision

Inelastic collision is when the colliding bodies stay together and move with the same velocity after collision.

In short, for inelastic collision,

Total momentum before

collision= Total momentum

 $m_1u_1+m_2u_2=m_1v_1+m_2v_2$

<u>Comparisons between Elastic collision and Inelastic collision</u>

Elastic collision	Inelastic collision
(i) Bodies separate after collision	Bodies stick together after collision.
(ii) Bodies move with different velocities after collision	Bodies move with same velocity after collision
(iii) Kinetic energy of the bodies is conserved	Kinetic energy of the bodies is not conserved
Momentum is conserved	Momentum is conserved

NOTE; For any stationary body or body at rest, the initial velocity is zero so the initial momentum of such a body before collision is zero.

Total momentum before

momentum after collision

 $m_1 u_1 + m_2 u_2 = (m_1 + m_2) V$

Total

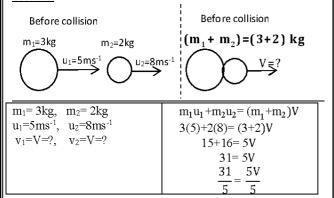
collision=

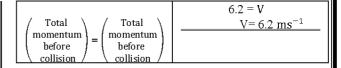
Example:1

after collision

A body of mass 3kg traveling at 5ms⁻¹ collides with a 2kg body moving at 8ms⁻¹ in the same direction. If after collision the two bodies moved together, Calculate the velocity with which the two bodies move after collision.

Solution

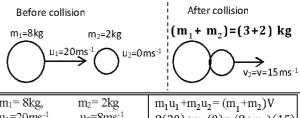


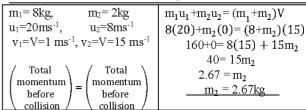


Example: 2

A body of mass 8kg traveling at 20 ms⁻¹ collides with a stationary body and they both move with velocity of 15ms⁻¹. Calculate the mass of the stationary body.

Solution

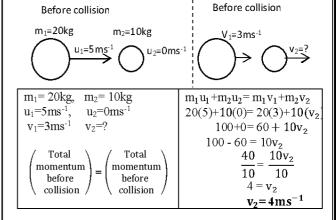




Example: 3

A body of mass 20kg traveling at 5ms⁻¹ collides with another stationary body of mass 10kg and they move separately in the same direction. If the velocity of the 20kg mass after collision was 3ms⁻¹. Calculate the velocity with which the 10kg mass moves.

Solution



Exercise:

- 1. A particle of mass 200g moving at 30ms⁻¹ hits a stationary particle of mass100g so that they stick and move together after impact. Calculate the velocity with which they move after collision.(Ans:V=20ms⁻¹)
- 2. A military tanker of mass 4tonnes moving at 12ms⁻¹ collides head on with another of mass 3tonnes moving at 20ms⁻¹. After collision, they stick together and move as one body. Ignoring the effect of friction, find their common velocity.

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(Ans: V=1.7ms⁻¹in the direction of the 2nd tank)

- 3. A body of mass 10kg moving at 20ms⁻¹ hits another body of mass 5kg moving in the same direction at 10ms⁻¹. After collision, the second body moves separately forward with a velocity of 30ms⁻¹. Calculate the velocity of the first body after collision. .(Ans:v₁=10ms⁻¹)
- 4. A car X of mass 1000kg travelling at a speed of 20 ms-I in the direction due east collides head-on with another car Y of mass 1500kg, travelling at 15ms-1 in the direction due west. If the two cars stick together, find their common velocity after collision.

EXPLOSIONS

Momentum is conserved in explosions such as when a riffle is fired. During the firing, the bullet receives an equal but opposite amount of momentum to that of the rifle.

Total momentum before collision = Total momentum after collision

$$\begin{split} m_{\mathbf{g}}\mathbf{u}_{\mathbf{g}} + m_{\mathbf{b}}\mathbf{u}_{\mathbf{b}} &= m_{\mathbf{g}}\mathbf{v}_{\mathbf{g}} + m_{\mathbf{b}}\mathbf{v}_{\mathbf{b}} \\ m_{\mathbf{g}}(0) + m_{\mathbf{b}}(0) &= m_{\mathbf{g}}\mathbf{v}_{\mathbf{g}} + m_{\mathbf{b}}\mathbf{v}_{\mathbf{b}} \\ 0 &= m_{\mathbf{g}}\mathbf{v}_{\mathbf{g}} + m_{\mathbf{b}}\mathbf{v}_{\mathbf{b}} \\ \mathbf{m}_{\mathbf{g}}\mathbf{v}_{\mathbf{g}} &= -\mathbf{m}_{\mathbf{b}}\mathbf{v}_{\mathbf{b}} \end{split}$$

Where; m_g is mass of the rifle (or gun), V_g is velocity of the rifle which is also called recoil velocity. m_b is mass of the bullet, V_b is velocity of the bullet.

For any explosion of bodies, the amount of momentum for one body is equal but opposite to that of another body.

The negative sign indicates that the momenta are in opposite directions.

Example:1

A bullet of mass 8g is fired from a gun of mass 500g. If the missile velocity of the bullet is 500ms⁻¹. Calculate the recoil velocity of the gun.

<u>Solution</u>

$$m_b = 8g = \frac{8}{1000} = 0.008kg,$$
 $m_g = 500g = \frac{500}{1000} = 0.5kg$
 $v_b = 500ms^{-1},$ $v_g = ?$

From,
$$m_g v_g = -m_b v_b$$

 $0.5 V_g = -0.008(500)$
 $0.5 V_g = -4$

$$\frac{0.5V_g}{0.5} = \frac{-4}{0.5}$$

$$V_g = -8ms^{-1}$$

The negative sign indicates that the recoil velocity, $V_{\rm g}$ is in opposite direction to that of the bullet.

Example:2

A bullet of mass 200g is fired from a gun of mass 4kg. If the muzzle velocity of the bullet is 400ms⁻¹, calculate the recoil velocity.

Solution

$$\begin{array}{l} m_b = 200g = \frac{200}{1000} = 0.2 kg, \quad m_g = 4 kg \\ v_b = 400 ms^{-1}, \qquad \qquad v_g = ? \end{array}$$

From,
$$m_g v_g = -m_b v_b$$

 $4V_g = -0.2(400)$
 $4V_g = -80$
 $\frac{4V_g}{4} = \frac{-80}{4}$
 $V_g = -20 \text{ms}^{-1}$

Example: 3

A bullet of mass 12.0g travelling at 150ms⁻¹penetrates deeply into a fixed soft wood and is brought to rest in 0.015s. Calculate

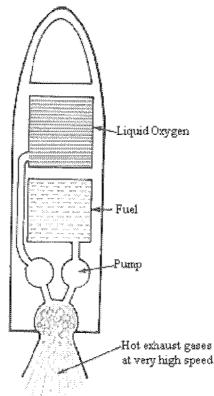
- (i) How deep the bullet penetrates the wood [1.125m]
- (ii) the average retarding force exerted by the wood on the bullet. [120N]

ROCKET AND JET ENGINES

These work on the principle that in any explosion one body moves with a momentum which is equal and opposite to that of another body in the explosion. For the rocket and the jet engine, the high velocity hot gas is produced by the burning of fuel in the engine.

Note: Rockets use liquid oxygen while jets use oxygen from air.

How a rocket engine work:



<u>Principle:</u> the jet and rocket engines work on the principle that momentum is conserved in explosion.

<u>High velocity</u>: the high velocity of the hot gas results in the burning of the fuel in the engine.

<u>Large momentum</u>: the large velocity of the hot gas results in the gas to leave the exhaust pipe with a large momentum.

<u>Engine:</u> the engine itself acquires an equal but opposite momentum to that of the hot gas.

Note: when the two bodies collide and they move separately after collision but in opposite directions then. $m_1u_1+m_2u_2=m_1v_1+m_2(-v_2)$ $m_1u_1+m_2u_2=m_1v_1-m_2v_2$

Example:

A body Q of mass 50g collides with a stationary body "P" of mass 4g. If a body "Q" moves backward with a velocity of $10 \, \mathrm{ms^{-1}}$ and a body "P", moves forward with a velocity of $6 \, \mathrm{ms^{-1}}$. Calculate the initial velocity of a body Q.

<u>Solution</u>

Total momentum before collision = Total momentum after collision

$$m_Q u_Q + m_P u_P = \ m_Q v_Q + m_p v_P$$

$$0.05\mathbf{u}_{\mathbf{Q}} + 0.004(0) = 0.05(-10) + 0.004(6)$$

$$0.05\mathbf{u}_{\mathbf{Q}} = -0.5 + 0.024$$

$$0.05\mathbf{u}_{\mathbf{Q}} = -0.476$$

$$\frac{0.05\mathbf{u}_{\mathbf{Q}}}{0.05} = \frac{-0.476}{0.05}$$

$$\mathbf{u}_{\mathbf{Q}} = -9.52\mathbf{m}\mathbf{s}^{-1}$$

Thus, the initial velocity of Q is 9.52ms⁻¹ to the left

Example: 2

A moving ball "P" of mass 100g collides with a stationary ball Q of mass 200g. After collision, P moves backward with a velocity of 2ms⁻¹ while Q moves forward with a velocity of 5ms⁻¹. Calculate the initial velocity of P.

Solution

Total momentum before collision = Total momentum after collision

$$m_{Q}u_{Q}^{}+m_{P}u_{P}^{}=m_{Q}v_{Q}^{}+m_{p}^{}v_{P}^{}$$

$$0.2(0) + 0.1u_p = 0.2(5) + 0.1(-2)$$

$$0.1u_{Q} = 1 + -0.2$$

$$0.1u_{Q} = -0.8$$

$$0.1u_{Q}$$

$$0.1$$

$$0.1$$

$$u_{P} = -8ms^{-1}$$

Thus, the initial velocity of Q is 8ms⁻¹ towards Q.

Example: 3.

A body of mass 10kg moves with a velocity of 20ms⁻¹. Calculate its momentum.

Solution

$$\overline{m=10\text{kg}}; \ v=20\text{ms}^{-1}$$
 Linear Momentum = Mass \times Velocity = 10×20 = 200kgm^{-1}

$$\begin{array}{ll} \mbox{Initial Momentum} & = \mbox{Mass} & \times \mbox{Initial Velocity} \\ & = \mbox{mu} \end{array}$$

Example:2

A 20kg mass traveling at 5mls is accelerated to 8mls. Calculate the change in momentum of the body.

Solution

Solution	
m=10kg	
u=5ms ⁻¹	
v=8ms ⁻¹	
Initial Momentum = mu	Final Momentum
= 20×5	= mv
$= 100 \text{kgms}^{-1}$	= 20×8
	= 160kgms ⁻¹
Change in Momentum = mv - mu	
= 160 - 100	
$=60kgms^{-1}$	

Note: The change in momentum is called Impulse.

Example:3

A one tonne car traveling at 20ms⁻¹ is accelerated at 2ms⁻² for five second. Calculate the;

- (i) change in momentum
- (ii) rate of change in momentum
- (iii) Accelerating force acting on the body.

Solution

m=1tonne =1000kg	
u=20ms ⁻¹	(ii) Rate of change in
v=?	momentum
a=2ms ⁻²	
t= 5s	Rate of change in momentum
(i) change in momentum	Change in mometum
Change in Momentum	Time taken
= mv - mu	
	_m(v-u)
=m(v-u)	t
But; $v = u + at$	
v = 20 + 2(5)	_ 1000(30-20)
$v = 30ms^{-1}$	_ 5
	_ 10000
Change in Momentum	5

= mv - mu	=2000 N
=m(v-u)	
=1000(30-20)	
=1000(10)	
$= 10,000 \mathrm{kgms^{-1}}$	

NOTE: The S.I unit for the rate of change in momentum is a newton.

(iv) Accelerating force acting on the body.

Accelerating force, F = Rate of change in momentum

$$= \frac{m(v-u)}{t}$$

$$= \frac{1000(30-20)}{5}$$

$$F = 2000N$$

From above, the force applied is equal to the rate of change in momentum. This leads to Newton's second law of motion.

Exercise

- 1. A body of mass 600g traveling at 10mls is accelerated uniformly at 2 ms⁻² for four seconds. Calculate the;
 - (i) change in momentum
 - (ii) force acing on a body

Solution

3	<u>olution</u>	
	(i)	(ii)
	mass, m = 600g	Rate of change in momentum
	600	m(v-u)
	$=\frac{1000}{1000}$	=
	= 0.6kg	0.6(18-10)
	u=10ms ⁻¹	4
	v=?	4.8
	a=2 ms ⁻²	4
	t = 4s	<u>=1.2 N</u>
	From;	
	v = u + at	(iii) Force acting on the body
	v = 10 + 2(4)	But; $v = u + at$
	$v = 18ms^{-1}$	But; $18 = 10 + 4t$
	Change in Momentum	But; $a = 2 \text{ms}^{-1}$
	= mv - mu	
		F = ma
	= m(v-u)	=0.6(2)
		F = 1.2 N
	= 0.6(18 - 10)	
	ĺ	Thus, Force acting on the body
	= 0.6(8)	is equal to the rate of change in
	$= 4.8 k gms^{-1}$	momentum.

Evample:4

A van of mass 1.5 tonnes travelling at 20ms⁻¹, hits a wall and is brought to rest as a result in 0.5seconds. Calculate the:

- (i) Impulse
- (ii) Average force exerted on the wall.

Solution	
m=1.5tonnes	(i)Impulse:
$=1.5 \times 1000$	Impulse = Change in Momentum
= 1500	= mv - mu
u=20ms ⁻¹	
v=0ms-1	= m(v - u)
t=0.5s	, ,
	= 1500(0-20)
	, ,
	= 1500(-20)
	$=-30,000kgms^{-1}$

The Negative sign means that the direction of the impulse is opposite to that in which the van was moving.

```
(ii)Average force exerted on the wall:

From;Impuse = Force \times Time = Ft

-30000 = F \times 0.5

F = -60,000N
```

Example:5

0 -1--4: ---

A man of mass 60kg jumps from a high wall and lands on a hard floor at a velocity of 6ms. Calculate the force exerted on the man's legs if;

- (i) He bends his knees on landing so that it takes 1.2s for his motion to be stopped.
- (ii) He does not bend his knees and it takes 0.06s to stop his motion.

Solution

SOLUCION	
(i)	
m=60kg	(ii)
u=6ms ⁻¹	m= 60kg
v=0ms ⁻¹	u=6ms ⁻¹
t=1.2s	v=0ms ⁻¹
	t=0.06s
Force acting on the	
body	Force acting on the body
But; $v = u + at$	But; $v = u + at$
But; $0 = 6 + 1.2a$	But; $0 = 6 + 0.06a$
But; $a = -5 \text{ms}^{-1}$	But; $a = -100 \mathrm{ms}^{-1}$
F = ma	F = ma
= 60(-5)	= 60(-100)
F = -300 N	F =-6000 N

Note

- The negative signs means the force acts to oppose that exerted by the man.
- Landing in (ii) exerts a larger force on the knees, which can cause injury compared to that in (ii).

Exercise

- 1. An athlete of 80 kg moving at 5ms⁻¹, slides trough a distance of 10m before stopping in 4 seconds. Find the work done by friction on the athlete.
- **2.** A car of mass 1500kg starts from rest and attains a velocity of 100ms⁻¹in 20 seconds. Find the power developed by the engine.
 - A. 750kW B. 3,000kW C. 30, 000kW D. 750, 000kW
 - 3. A ball of 3kg moves at 10ms⁻¹ towards a volley ball player. If the player hits the ball and the ball moves